Performance Analysis of Fractional Order PID Controller with the Conventional PID Controller for Bioreactor Control

Shivaji Karad, Dr. S. Chatterji, Prasheel Suryawanshi

Abstract— Despite the dramatic advancement of process control in recent decades, the proportional-integral-derivative (PID) controller continues to be the most frequently used feedback controller today. PID control mechanism, the ubiquitous availability of reliable and cost effective commercial PID modules, and pervasive operator acceptance are among the reasons for the success of PID controllers. An elegant way of enhancing the performance of PID controllers is to use fractional-order controllers where I and D-actions have, in general, non-integer orders. In a PIλDδ controller, besides the proportional, integral and derivative constants, denoted by Kp, Ti and Td respectively, we have two more adjustable parameters: the powers of s in integral and derivative actions, viz. λ and δ respectively. This paper compares the performance of conventional PID and fractional PID controllers used for bio-reactor control.

Index Terms — PID controller, Fractional Calculus, Fractional PID controller, Bio-reactor control

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1 INTRODUCTION

The PID controllers have remained, by far; the most commonly used in practically all industrial feedback control applications. The main reason is its relatively simple structure, which can be easily understood and implemented in practice. They are thus, more acceptable than advanced controllers in practical applications unless evidence shows that they are insufficient to meet specifications. Many techniques have been suggested for their parameters tuning. Although all the existing techniques for the PID controller parameter tuning perform well, a continuous and an intensive research work is still underway towards system control quality enhancement and performance improvements. On the other hand, in recent years, it is remarkable to note the increasing number of studies related with the application of fractional controllers in many areas of science and engineering. This fact is due to a better understanding of the fractional calculus potentialities. In the field of automatic control, the fractional order controllers which are the generalization of classical integer order controllers would lead to more precise and robust control performances. Although it is reasonably true that the fractional order models require the fractional order controllers to achieve the best performance, in most cases the researchers consider the fractional order

controllers applied to regular linear or non-linear dynamics to enhance the system control performances.

This paper is organized as follows: In section 2, we present a brief introduction to fractional calculus. Section 3 deals with fractional order PID controller. Section 4 presents the application of proposed fractional PID controller for bio-reactor control system. Section 5 deals with simulation results of the system and section 6 discuss the conclusion.

2 FUNDAMENTALS OF FRACTIONAL CALCULUS

2.1 Definitions of Fractional Calculus

Fractional calculus is an old mathematical topic since 17th century. Fractional calculus is a subdivision of calculus theory which generalizes the derivative or integral of a function to non-integer order. Fractional calculus helps evaluating $(d^n y/dt^n)$, n-fold integrals where n is fractional, irrational or complex. For fractional order systems n is considered to be fractional. The number of applications where fractional calculus has been used rapidly grows. These mathematical phenomena allow to describe a real object more accurately than the classical "integer-order" methods. The real objects are generally fractional however, for many of them the fractionality is very low. The main reason for using the integer-order models was the absence of solution methods for fractional differential equations. At present there are lots of methods for approximation of fractional derivative and integral and fractional calculus can be easily used in wide areas of applications (e.g.: control theory - new fractional controllers and system models, electrical circuits theory - fractances, capacitor theory, etc.) [1], [2].

The generalized fundamental operator which includes the differentiation and integration is given as:

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$$aD_t^q = \begin{cases} \dfrac{d^q}{dt^q} & , R\ (q) > 0 \\[2mm] 1 & , R\ (q) = 0 \qquad (1) \\[2mm] \int_a^t (d\tau)^{-q} & , R\ (q) < 0 \end{cases}$$

Where,

      a – Lower limit of integration

      t – Upper limit of integration

      q – Order of fractional differentiation or integration

q with negative value indicates integration while q with positive value indicates differentiation [2].

The theory of fractional-order derivative was developed mainly in the 19th century. There are several definitions of fractional order derivative. Two important and widely applied definitions are:

Grunwald-Letnikov definition is perhaps the best known due to its most suitability for the realization of discrete control algorithms. The Grunwald-Letnikov definition is expresses as:

$$aD_t^\alpha f(t) = \lim_{h \to 0} h^{-\alpha} \sum_{j=0}^{\left[\frac{t-a}{h}\right]} (-1)^j \binom{\alpha}{j} f(t - jh) \qquad (2)$$

Where,   [x] is the integer part of x

           h is the time step.

The Riemann-Liouville definition is expresses as:

$$f(t) = \frac{d^n}{dt^n} \int_a^t \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau \qquad (3)$$

For a wide class of functions which appear in real physical and engineering applications, the Riemann-Liouville and the Grunwald-Letnikov definitions are equivalent [2].

2.2 Fractional Order Systems

Feedback control system is one of the major areas where the concept of fractional calculus is being applied to obtain an efficient system and also longevity and freedom. To the control engineer to compensate any shifts in the transfer function due to parametric spreads, aging etc. a system is efficient if the controller is of the similar order to that of a plant being controlled. In reality the systems are of fractional order and therefore to have a fractional order controller will be efficient [1, 4].

The time domain representation of a fractional order system is given as:

$$a_n D^{\alpha n} y(t) + a_{n-1} D^{\alpha n-1} y(t) + \ldots + a_0 D^{\alpha 0} y(t)$$

$$= b_m D^{\beta m} u(t) + b_{m-1} D^{\beta m-1} u(t) + \ldots + b_0 D^{\beta 0} u(t) \qquad (4)$$

The transfer function of the system is:

$$G(s) = \frac{b_m S^{\beta m} + b_{m-1} S^{\beta m-1} + \ldots + b_0 S^{\beta 0}}{a_n S^{\times n} + a_{n-1} S^{\times n-1} + \ldots + a_0 S^{\times 0}} \qquad (n \geq m) \qquad (5)$$

Where,

β_k (k = 0,1,…..,m), α_k (k = 0,1,…..,n) are arbitrary real numbers

a_k (k = 0,1,….,n) and b_k (k = 0,1,…….,m) are arbitrary constant.

The feedback control loop of a fractional order system with a fractional controller is similar to the integer order feedback control loop. It is shown in Fig. 1
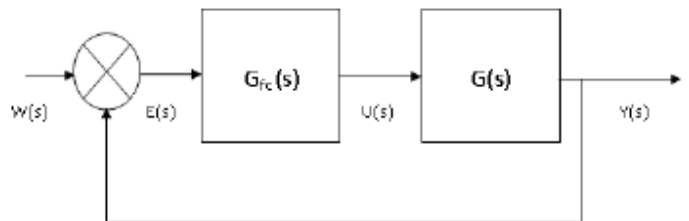

Fig. 1 Feedback Control Loop for a Fractional System

Where,

      W(s) – Input Signal

      E(s) – Error Signal

      G_{fc}(s) – Controller Transfer Function

      G(s) – System or plant Transfer Function

      Y(s) – output Signal

      U(s) – Controller Signal

3 FRACTIONAL ORDER PID CONTROLLER

PID controllers belong to dominant industrial controllers and therefore are topics of steady effort for improvements of their quality and robustness. One of the possibilities to improve PID controllers is to use fractional-order controllers with non-integer derivation and integration parts.

A fractional order PID controller is represented as $PI^\lambda D^\delta$. It allows us to adjust derivative (λ) and integral (δ) order in addition to the proportional, integral and derivative constants where the values of λ and δ lie between 0 and 1. This gives extra freedom to operator in terms of two extra knobs i.e.

-    Order of differentiation

-    Order of integration

This also provides more flexibility and opportunity to better adjust the dynamical properties of the control system. The fractional order controller revels good robustness. The robustness of fractional controller gets more highlighted in presence of a non-linear actuator. Fig. 2 shows the concept of a fractional PID control system [5].
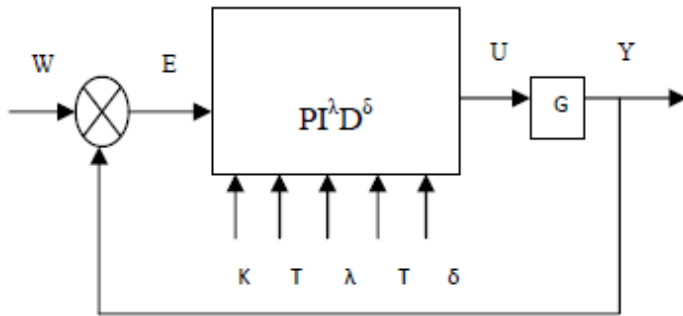

Fig. 2 Fractional PID Control System

The integro-differential equation defining the control action of a fractional order PID controller is given by:

$$K_pe(t) + T_i{}^{-\lambda}e(t) + T_dD^\delta e(t) = u(t) \qquad (6)$$

Applying Laplace transform to this equation with null initial conditions, the transfer function of the controller can be expressed by:

$$G_{fc} = K_p+T_is^{-\lambda}+T_ds^\delta \qquad (7)$$

Taking $\lambda=1$ and $\delta=1$, we can obtain a classical PID controller. If $\lambda=0$ we obtain a PD^δ controller, etc. All these types of controllers are the particular cases of the $PI^\lambda D^\delta$ controller. Actually, in theory, $PI^\lambda D^\delta$ itself is an infinite dimensional linear filter due to the fractional order in differentiator or integrator.

It can be expected that $PI^\lambda D^\delta$ controller may enhance the system control performance due to more tuning knobs introduced. Actually, in theory $PI^\lambda D^\delta$ itself is an infinite dimensional linear filter due to the fractional order in differentiator or integrator.

4 BIO-REACTOR CONTROL SYSTEM

Biochemical reactors are cylindrical culture vessels used for the fermentation process in which anaerobic breakdown of complex organic materials by the action of anaerobic microorganism or free enzymes takes place. Materials such as carbon, nitrogen, oxygen, which are called substrate, and other nutrients are brought with the cell into the culture vessel (bioreactor) and converted within the cell via hundreds of reactions to the various constituents of the cell as well as to biochemical product. Bioreactors provide a controlled environment that is necessary to bring the better growth of microbes, and also maintain constant temperature according to

the need of microbes. A typical control and instrumentation diagram of the bioreactor used for analysis, with biomass concentration as the measured output, is shown in Fig. 3.
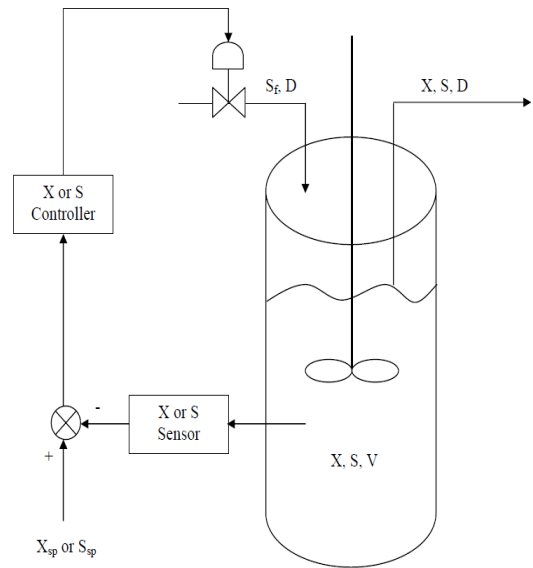

Fig. 3 Simplified PI diagram for a Bio-reactor

In the following a bio-reactor system is used to evaluate the control performance of proposed Fractional PID controller. The performance of the fractional PID controller is compared with the performance of conventional PID controller, which is widely used to control the chemical processes.

In the present control study the parameters used for the Monod model are:

μ_{max} = 0.53 h^{-1},
Ks   = 0.12 g/l,
Yx/s = 0.4,
Yp/x = 0.5,
Sf = 4.0 g/l,

The non-linear process has the following steady-state for a dilution-rate of 0.43 h^{-1} at which the production rate is maximum,

Bio-mass-concentration, X = 1.3936 g/l

Substrate-concentration, S = 0.5160 g/l

Product-concentration, P = 0.6968 g/l

The maximum-production rate is given by:

$$D_{maxoutput} = \mu_{max}\left[1 - \sqrt{\frac{Ks}{Ks+Sf}}\right] \qquad (8)$$

The process is controlled at this operating point.

The state-space formulation is used to linearize the non-linear equations around the steady-state operating-point [10]. The

transfer function relating the dilution-rate to the biomass concentration is:

$$G_p(s) = \frac{-1.39s^2 - 1.99s - 0.2577}{s^3 + 1.408s^2 + 0.656s + 0.1013} \qquad (9)$$

5 SIMULATION RESULTS AND DISCUSSION

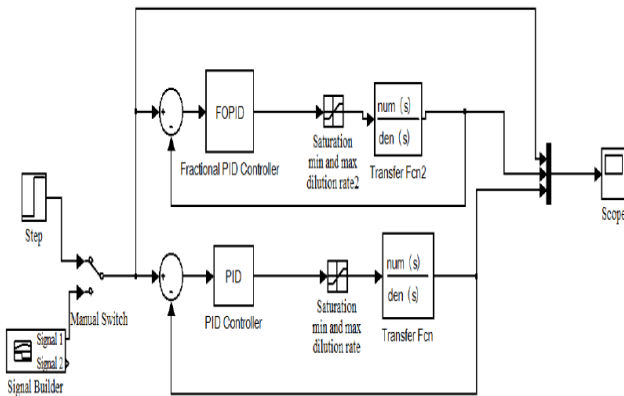Fig.4 shows the Simulink model for bioreactor control using PID and FOPID controllers.

Fig.4 Simulink model for bio-reactor control using PID and FOPID controller

A PID controller is design for the integer order model of bio-reactor using MATLAB PID tuner block. The system (Fig. 3) is tuned for the following parameters for the PID controller:

$K_p = -2.7$

$K_i = -1.0$

$K_d = 0.7$

Fig.5 shows the time response characteristics for a step change of bioreactor control system and Table 1 shows its time response specifications.
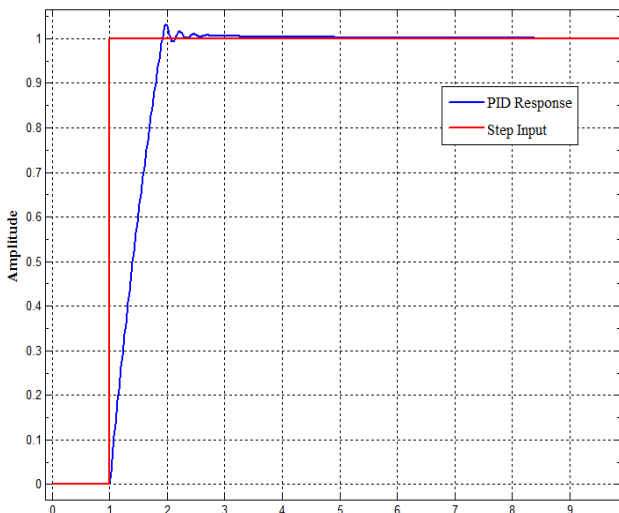
Specifications	Value
Overshoot	4%
Rise- time	0.8 hr
Settling- time	5 hr
Steady state value	1

Fig. 5 Time response of Bio-reactor using PID Cotroller

TABLE 1 TIME RESPONSE CHARACTERISTICS OF THE SYSTEM REPRESENTED IN FIG. 5

Parameters	Value
Overshoot	5%
Rise-time	½ hours
Settling-time	8 hours
Steady-state value	1

PID controller works well for the system with fixed parameters. However, in the presence of large parameter variations or major external disturbances, the PID controllers usually face trade-off between:

- Fast response with significant overshoot.
- Smooth but slow response.

Bio-reactor control system is a highly complex, non- linear and uncertain system. For such systems a controller with more number of tuning parameters and which works well for the complex non-linear systems is to be used.

Fractional PID controller is one such controller which is to be design for the bio-reactor control. A fractional PID controller is designed for the system by experimental method with following parameters:

$K_p = -2.7$

$K_i = -0.8, \lambda = 1$

$K_d = 1, \quad \delta = 0.8$

Fig.6 shows the time response characteristics and Table 2 shows its time response specifications.

TABLE 2 TIME RESPONSE CHARACTERISTICS OF THE SYSTEM REPRESENTED IN FIG. 6
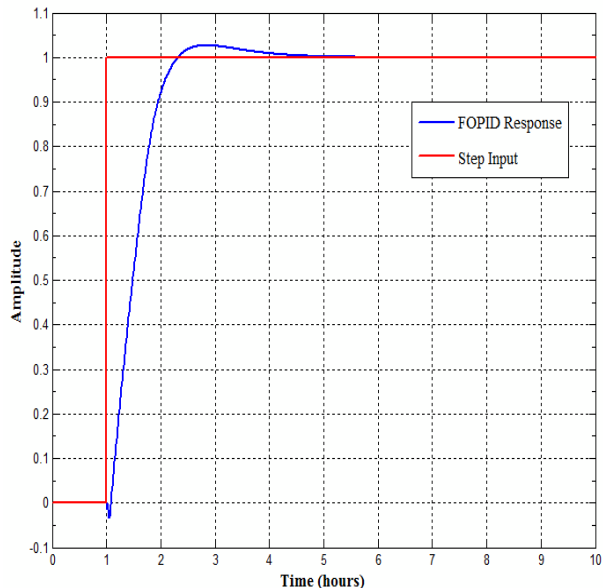
instant negative peak overshoot.

Fig. 6 Time response of Bio-reactor using FOPID Controller

Fig. 7 shows the simulation results for the bio-reactor control using both, the conventional PID controller and the Fractional order PID controller.
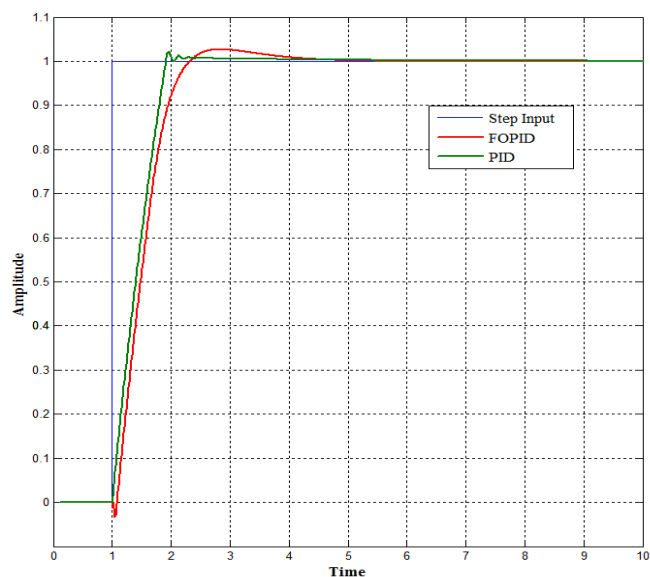

Fig. 7 Step Response of PID vs FOPID for Bio-reactor

Fig. 8 below shows the set-point change tracking response for both the PID and FOPID controller. It is observed that FOPID controller shows a more peak overshoot than the PID controller but it's settling time for the set-point change is very less as compared to PID controller. Also due to the non-minimal phase system the FOPID controller gives a small
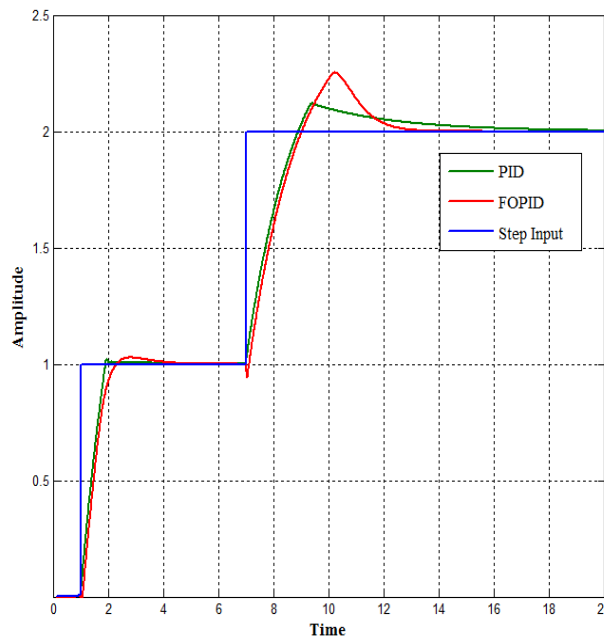

Fig. 8 Set-point tracking response for PID and FOPID controller

The PID and fractional order PID controllers are designed for the bio-reactor control process. The characteristics are compared in table 3.

TABLE 3 COMPARISION OF TIME RESPONSE CHARACTE-RISTICS OF PID CONTROLLER AND FRACTIONAL PID CONTROLLER

Characteristics	PID Controller	FOPID Controller
Peak Value	5%	4%
Settling- Time	8	5
Rise- Time	0.5	0.8
Steady State Value	1	1

6 CONCLUSION

In present work, performance comparison of a conventional PID controller with that of fractional order PID controller has been presented. Firstly, a simulation model of bio-reactor control system is constructed with the help of MatLab/Simulink module. Then, performance comparison of PID controller with

that of the fractional order PID controller are simulated and studied. Comparing the step responses with the ones obtained (in simulation) with the PID controller, the better performance of the system with the fractional order PID controller was observed.

Fractional order PID controller for integer order system offer better flexibility in adjusting gain and phase characteristics than the PID controllers, owing to the two extra tuning parameters i.e. order of integration and order of derivative in addition to proportional gain, integral time and derivative time.

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